$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Difference Quotient

[Derivative of $f(x)$

Slope at a point]

Derivative – the derivative of a function f with respect to the variable x is the function f' whose value at x is $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$, provided the limit exists.

Derivative at a point – the derivative of a function f at the point $x = a$ is the limit

$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$, provided the limit exists.

Notation

$$f'(x),\ \frac{dy}{dx},\ y',\ \frac{df}{dx},\ \frac{d}{dx}f(x),\ D_x[y]$$

$f(x)=\sqrt{x}$ $\quad \frac{f(x+h)-f(x)}{h}$

$$\frac{\sqrt{x+h}-\sqrt{x}}{h}\cdot\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}$$

$$\frac{x+\cancel{h}-x}{\cancel{h}(\sqrt{x+h}+\sqrt{x})} \to \frac{1}{\sqrt{x+h}+\sqrt{x}} \quad \lim_{h\to 0}\frac{1}{\sqrt{x+0}+\sqrt{x}} \to \frac{1}{2\sqrt{x}}$$

$$\frac{\sqrt{x}-\sqrt{a}}{x-a}$$

$$\frac{\sqrt{x}-\sqrt{a}}{x-a}\cdot\frac{\sqrt{x}+\sqrt{a}}{\sqrt{x}+\sqrt{a}}$$

$$\frac{x-a}{(x-a)(\sqrt{x}+\sqrt{a})}$$

$$\lim_{x\to a}\frac{1}{\sqrt{x}+\sqrt{a}}=\frac{1}{2\sqrt{a}}$$

EXAMPLE 3 GRAPHING f' from f

Graph the derivative of the function f whose graph is shown in Figure 3.3a. Discuss the behavior of f in terms of the signs and values of f'.

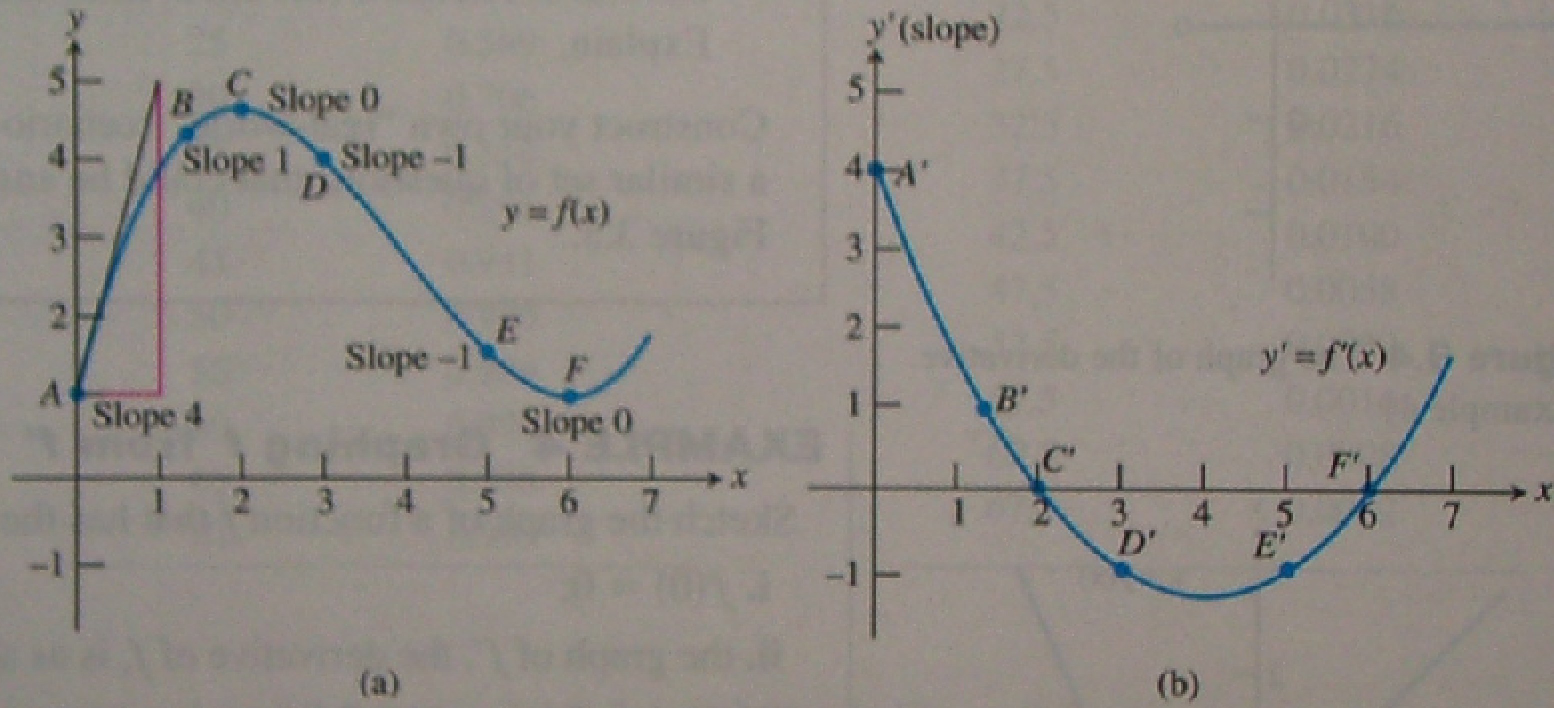

Figure 3.3 By plotting the slopes at points on the graph of $y = f(x)$, we obtain a graph of $y' = f'(x)$. The slope at point A of the graph of f in part (a) is the y-coordinate of point A' on the graph of f' in part (b), and so on. (Example 3)

Sect. 3.1

Q.R. #7, 9, 10

P.S. #1-4, 5, 7, 9, 13-19, 21, 23, 25, 26, 28